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FIXED POINT RESULT IN DOUBLE CONTROLLED METRIC- LIKE SPACES

S. S. P. Singh

Assistant Professor, Department of Mathematics, S.N.SinhaCollege, Warisaliganj, Nawada, Bihar, India

ABSTRACT

In this paper, we obtain a fixed point result in a complex-valued double controlled metric-like space and provide an example in support of our result

KEYWORDS: *b-Metric Space, Extended b- Metric Space, Controlled Metric Space, Double Controlled Metric Space.*

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INTRODUCTION

Banach's contraction principle has long been one of the most important tools in the study of nonlinear problems, and the Banach fixed point theorem has numerous applications both inside and outside mathematics. Bakhtin [1] introduced the concept of a *b-metric space* and established several fundamental results, which were later generalized by many other researchers (see [2, 3]). Kamran et al. [4] and others extended the notion of *b-metric spaces* by controlling the triangle inequality rather than using a control function in the contractive conditions.

Ullah et al. [5] introduced the concept of *complex-valued extended b-metric spaces* and obtained some fixed point results. Mlaiki et al. [6] and Abdeljawad et al. [7] proposed *double controlled metric-type spaces* and established the Banach contraction principle in this setting. The classical definition of a metric space was generalized by Harandi [8], who introduced the notion of a *metric-like space*. Mlaiki [9] and Aysegul [10] further extended this idea by defining *double controlled metric-like spaces* and proving related fixed point theorems.

Azam [11] introduced the *complex-valued metric space*, while Panda [12] proposed the *complex-valued double controlled metric space*. Hosseini and Karizaki [13] generalized Panda's results by introducing the *complex-valued metric-like space*. Chowdhary et al. [14] further developed this line of research by defining the *complex-valued double controlled metric-like space*.

Recently, Souayah and Hidri [15] proved a fixed point theorem for *Caristi contraction mappings* in controlled metric spaces. In this paper, we establish a fixed point result in the framework of *complex-valued double controlled metric-like spaces* using Caristi contraction mappings. We also provide an illustrative example to support our findings. Our results generalize those of Souayah and Hidri [15] and several others.

PRELIMINARIES

Let us recall some definitions, useful in the introductions of our concept.

Let C be the set of complex numbers and $w_1, w_2 \varepsilon C$. $w_1 \leq w_2$ if and only if

$Re(w_1) \leq Re(w_2)$ or $Re(w_1) = Re(w_2)$ and $Im(w_1) \leq Im(w_2)$.

Taking into account the previous definition, we have that $w_1 \leq w_2$ if one of the next conditionsis satisfied:

- $Re(w_1) < Re(w_2)$ and $Im(w_1) < Im(w_2)$;
- $Re(w_1) < Re(w_2)$ and $Im(w_1) = Im(w_2)$;
- $Re(w_1) < Re(w_2)$ and $Im(w_1) > Im(w_2)$;
- $Re(w_1) = Re(w_2)$ and $Im(w_1) < Im(w_2)$;

Definition 2.1 [1]

Let $\$ \neq \phi$ and $\mho \geq 1$be a given real number. Let £: $\$ x \$ \rightarrow [0,+\infty)$ be a function is called b- metric if

- £(p, q) ≥ 0, `
- £(p, q) = 0 if and only if p = q,
- £(p, q) = £(q, p),
- £(p, q) ≤ $\mho$ [£(p, g) + £(g, q)] ∀ p, q, g ε \$.

A pair (\$, £) is called a **b-metric space**. Clearly, every metric space is a b-metric space (with s=1), but in general, a b-metric space is a proper extension of the usual metric space.

Definition 2.2 [5]

Let $\$ \neq \phi$ and given a function $\theta : \$ x \$ \rightarrow [1,+\infty)$. Let £ : $\$ x \$ \rightarrow C$ be a function is called complex - valued extended b- metric if the following conditions are satisfied

- £(p, q) ≥ 0, `
- £(p, q) = 0 if and only if p = q,
- £(p, q) = £(q, p),
- £(p, q) ≤ θ(p,q)[£(p, g) + £(g, q)] ∀ p, q, g ε \$.

A pair (\$, £) is called an complex- valued extended b-metric space.

Definition 2.3 [6]

Let $\$ \neq \phi$ and given a function $\theta : \$ x \$ \rightarrow [1,+\infty)$. Let £ : $\$ x \$ \rightarrow [0,+\infty)$ be a function is called controlled metric if

- £(p, q) ≥ 0, `
- £(p, q) = 0 if and only if p = q,
- £(p, q) = £(q, p),

- $\pounds(p, q) \le \theta(p,g)\pounds(p, g) + \theta(g,q)\pounds(g, q)] \ \forall\ p, q, g \ \varepsilon\ \$$.

 A pair $(\$, \pounds)$ is called a controlled metric space.

Definition 2.4[7]

Let $\$ \neq \phi$ and given a function $\theta,\varphi : \$ \text{ x } \$ \rightarrow [1,+\infty)$. Let $\pounds : \$ \text{ x } \$ \rightarrow [0,+\infty)$ be a function .is called double controlled metric if

- $\pounds(p, q) \ge 0$, `
- $\pounds(p, q) = 0$ iff $p = q$,
- $\pounds(p, q) = \pounds_2(q, p)$,
- $\pounds(p, q) \le \theta(p,g)\pounds(p, g) + \varphi(g,q)\pounds(g, q)] \ \forall\ p, q, g \ \varepsilon\ \$$.

 A pair $(\$, \pounds)$ is called a double controlled metric space.

Definition 2.5 [9]

Let $\$ \neq \phi$ and given a function $\theta,\varphi : \$ \text{ x } \$ \rightarrow [1,+\infty)$. Let $\pounds : \$ \text{ x } \$ \rightarrow [0,+\infty)$ be a function .is called double controlled metric- like if

- $\pounds(p, q) \ge 0$, `
- $\pounds(p, q) = 0$ implies $p = q$,
- $\pounds(p, q) = \pounds_2(q, p)$,
- $\pounds(p, q) \le \theta(p,g)\pounds(p, g) + \varphi(g,q)\pounds(g, q)] \ \forall\ p, q, g \ \varepsilon\ \$$.

 A pair $(\$, \pounds)$ is called a double controlled metric-like space.

Definition 2.6 [12]

Let $\$ \neq \phi$ and given a function $\theta,\varphi : \$ \text{ x } \$ \rightarrow [1,+\infty)$. Let $\pounds : \$ \text{ x } \$ \rightarrow C$ be a function .is called complex -valued double controlled metric if

- $\pounds(p, q) \ge 0$, `
- $\pounds(p, q) = 0$ iff $p = q$,
- $\pounds(p, q) = \pounds_2(q, p)$,
- $\pounds(p, q) \le \theta(p,g)\pounds(p, g) + \varphi(g,q)\pounds(g, q)] \ \forall\ p, q, g \ \varepsilon\ \$$.

A pair $(\$, \pounds)$ is called a complex- valued double controlled metric space.

Definition 2.7[14]

Let $\$ \neq \phi$ and given a function $\theta,\varphi : \$ \text{ x } \$ \rightarrow [1,+\infty)$. Let $\pounds : \$ \text{ x } \$ \rightarrow C$ be a function .is called complex -valued double controlled metric - like if

- £(p, q) ≥ 0, `
- £(p, q) = 0 implies p = q,
- £(p, q) = $£_2$(q, p),
- £(p, q) ≤ θ(p,g)£(p, g) + φ(g,q)£(g, q)] ∀ p, q, g ε $.

A pair($, £) is called a complex-valued double controlled metric-like space. Every complex-valued double controlled metric space is, in general, also a complex-valued double controlled metric-like space. However, the converse does not hold in general. Furthermore, the notion of a complex-valued double controlled metric-like space is a generalization of the concept of a complex-valued extended b-metric space.

Example 2.1[14]

Let $ = {1.2, 3}. Let complex- valued double controlled metric like £: $ x $ → C defined by

£(1, 1) = £(2, 2) = 0, £(3,3) = $\frac{i}{2}$, £(1,2)= £(2,1) = 2+4i,£(2,3)= £(3,2) = i, £(1,3)= £(3, 1) = 1- i

And θ, φ: $ x $ → [1,+ ∞) to be symmetry and defined by

θ(1,1) = θ(2,2) = θ(3,3) = 1, θ(1,2) = θ(2,1) = $\frac{6}{5}$, θ(2,3) = θ(3,2) = $\frac{8}{5}$, θ(1,3) = θ(3,1) = $\frac{151}{100}$.

φ(1,1) = φ(3,3) = φ(2,2) =1, φ(1,2) = φ(2,1) = $\frac{6}{5}$, φ(2,3) = φ(3,2) = $\frac{33}{20}$, φ(1,3) = φ(3,1) = $\frac{8}{3}$

Thus £ is complex- valued double controlled metric- like space.

Note that, $|£(2,1)| = \sqrt{20} >$ θ(2,1) $|£(2,3)|$ + θ(2,1) $|£(3,1)|$.Thus £ is not a complex – valued extended b-metric space for the function θ.

Again, $|£(2,1)| = \sqrt{20} >$ θ(2,3) $|£(2,3)|$ + θ(3,1) $|£(3,1)|$. Thus £ is not a complex- valued controlled metric- like space.

Definition 2.5 [14]

Let {p_σ} be a sequence in complex- valued double controlled metric – like space ($, £). Then

- {p_σ} is said to be convergent to p ε $ written as

  $\lim_{\sigma\to\infty}$£(p_σ, p) = 0.

- {p_σ} is said to be Cauchy sequence in $ written as

  $\lim_{\sigma,\tau\to\infty}$£($p_\sigma$, p_τ) = 0.

- ($, £) is said to be complete if every Cauchy sequence is a convergent sequence.

Definition 2.6 [14]

Let ($, £) be a complex- valued double controlled metric-like space . Let p ε $ and δ>0.

- The open ball B_p (p, δ) is

  B (p, δ) = { q ε $, £(p,q) < δ}.

- The mapping T: \$ → \$ is said to be continuous at ꝑ ε \$ if for all r > 0, there exists $\gamma > 0$ such that

  $T(B_p(ꝑ, \gamma))\ B_p(Tꝑ, r)$.

  Note that if t is continuous at p in (\$, £), then $p_\sigma \to p$ implies that $Tp_\sigma \to Tp$ when $\sigma \to \infty$.

MAIN RESULTS

Theorem 3.1

Let (\$, £) be a complex – valued double controlled metric- like space. Consider the function T : \$ → \$ such that

(3.1)£(Tꝑ, Tꝗ) ≤ (h(ꝑ) – h(Tꝑ))£(ꝑ, ꝗ) for all ꝑ,ꝗ ε \$.

Where h: \$ → R is a bounded function from below.

For, $ꝑ_o$ ε \$, take $ꝑ_\sigma = T^\sigma ꝑ_0$. Moreover, assume that, for every ꝑ ε \$, we have

(3.2) $\lim_{\sigma\to\infty}\theta(ꝑ_\sigma, ꝑ)$ and $\lim_{\sigma\to\infty}\varphi(ꝑ, ꝑ_\sigma,)$ exists and are finite and θ and φ satisfies the following conditions.

(3.3) $\mathrm{Sup}_{\tau\geq 1}\lim_{\sigma\to\infty}\theta(ꝑ_{i+1}, ꝑ_{i+2})\, \varphi(ꝑ_{i+1}, ꝑ_\tau) / \theta(ꝑ_i, ꝑ_{i+1}) < \frac{1}{k}$ where k ε (0,1).

Then T has a unique fixed point.

Proof

- ***Case 1***: Assume that there exists $\sigma \geq 0$, such that. $£(ꝑ_\sigma, Tꝑ_\sigma) = 0$, which implies that $ꝑ_\sigma = Tꝑ_\sigma$. then $ꝑ_\sigma$ is a fixed point of T.
- ***Case 2***: Assume that. $£(ꝑ_\sigma, Tꝑ_\sigma) > 0$ for all σ ε N. Let us denote $b_\sigma = £(ꝑ_{\sigma-1}, ꝑ_\sigma)$.

  From (3.1), we obtain

  $b_{\sigma+1} = £(ꝑ_\sigma, ꝑ_{\sigma+1}) = £(Tꝑ_{\sigma-1}, Tꝑ_\sigma) \leq (h(ꝑ_{\sigma-1}) - h(Tꝑ_{\sigma-1}))£(ꝑ_{\sigma-1}, ꝑ_\sigma) = (h(ꝑ_{\sigma-1}) - h(ꝑ_\sigma))£(ꝑ_{\sigma-1}, ꝑ_\sigma)$

  $= (h(ꝑ_{\sigma-1}) - h(ꝑ_\sigma))b_\sigma$

  (3.4) $0 < b_{\sigma+1} / b_\sigma \leq (h(ꝑ_{\sigma-1}) - h(ꝑ_\sigma))$, for all σ ε N.

  Therefore, the sequence $\{h(ꝑ_\sigma\}$ is required to be positive and non- increasing.

  Thus $\lim_{\sigma\to\infty} h(ꝑ_\sigma = r > 0$, now using (3.4),we obtain

  $\sum_{i=1}^{\sigma} b_{i+1} / b_i \leq \sum_{i=1}^{\sigma} h(ꝑ_{i-1}) - h(ꝑ_i) = h(ꝑ_0) - h(ꝑ_1) + h(ꝑ_1) - h(ꝑ_2) + h(ꝑ_2) - h(ꝑ_3) + \ldots + h(ꝑ_{\sigma-1}) - h(ꝑ_\sigma) = h(ꝑ_0) - h(ꝑ_\sigma)$

  Which means that $\sum_{i=1}^{\infty} b_{i+1} / b_i < \infty$. Consequently, we have

  (3.5) $\lim_{\sigma\to\infty} b_{i+1} / b_i = 0$.

  Taking into account (5), there exists i_0 ε N such that all $i \geq i_0$

  (3.6) $b_{i+1} / b_i = k$ for k ε (0,1).

  This gives that

  (3.7) $£(ꝑ_{\sigma+1}, ꝑ_\sigma) \leq k\, £(ꝑ_\sigma, ꝑ_{\sigma-1})$ for all $\sigma \geq \sigma_0$.

Now, we show that $\{p_\sigma\}$ is a Cauchy sequence. From (3.7), we obtain

(3.8) $£(p_{\sigma+1}, p_\sigma) \leq k^\sigma £(p_0, p_1)$ for all $\sigma \geq \sigma_0$.

For any $\sigma,\rho \in N$ $(\sigma < \rho)$,we have

$£(p_\sigma, p_\rho) \leq \theta(p_\sigma, p_{\sigma+1})£(p_\sigma, p_{\sigma+1}) + \varphi(_{\sigma+1}, p_\rho)£(p_{\sigma+1}, p_\rho)$

$\leq \theta(p_\sigma, p_{\sigma+1})£(p_\sigma, p_{\sigma+1}) + \varphi(_{\sigma+1}, p_\rho)[\theta(p_{\sigma+1}, p_{\sigma+2})£(p_{\sigma+1}, p_{\sigma+2}) + \varphi(_{\sigma+2}, p_\rho)£(p_{\sigma+2}, p_\rho)]$

$=\theta(p_\sigma, p_{\sigma+1})£(p_\sigma, p_{\sigma+1}) + \varphi(_{\sigma+1}, p_\rho)\theta(p_{\sigma+1}, p_{\sigma+2})£(p_{\sigma+1}, p_{\sigma+2}) + \varphi(_{\sigma+1}, p_\rho)\varphi(_{\sigma+2}, p_\rho)£(p_{\sigma+2}, p_\rho)$

$\leq \theta(p_\sigma, p_{\sigma+1})£(p_\sigma, p_{\sigma+1}) + \varphi(_{\sigma+1}, p_\rho)\theta(p_{\sigma+1}, p_{\sigma+2})£(p_{\sigma+1}, p_{\sigma+2}) + \varphi(_{\sigma+1}, p_\rho)\varphi(_{\sigma+2}, p_\rho)\theta(p_{\sigma+2}, p_{\sigma+3})£(p_{\sigma+2}, p_{\sigma+3}) + \varphi(p_{\sigma+1}, p_\rho)\varphi(p_{\sigma+2}, p_\rho)\varphi(p_{\sigma+3}, p_\rho)£(p_{\sigma+3}, p_\rho)$

$\leq \dots$

$\leq \theta(p_\sigma, p_{\sigma+1})£(p_\sigma, p_{\sigma+1}) + \sum_{i=\sigma+1}^{\rho-2}(\prod_{j=\sigma+1}^{i} \varphi(p_j, p_\rho)\theta(p_i, p_{i+1})£(p_i, p_{i+1})) + \prod_{k=\sigma+1}^{\rho-1} \varphi(p_k, p_\rho)£(p_{\rho-1}, p_\rho))$

$\leq \theta(p_\sigma, p_{\sigma+1})k^\sigma £(p_0, p_1) + \sum_{i=\sigma+1}^{\rho-2}(\prod_{j=\sigma+1}^{i} \varphi(p_j, p_\rho)\theta(p_i, p_{i+1})k^i £(p_0, p_1)) + \prod_{k=\sigma+1}^{\rho-1} \varphi(p_k, p_\rho)k^{\sigma-1}£(p_0, p_1))\dots$

$\leq \theta(p_\sigma, p_{\sigma+1})k^\sigma £(p_0, p_1) + \sum_{i=\sigma+1}^{\rho-1}(\prod_{j=o}^{i} \varphi(p_j, p_\rho)\theta(p_i, p_{i+1})k^i £(p_0, p_1))$

(3.9) Let $T_l = \sum_{i=0}^{l}(\prod_{j=o}^{i} \varphi(p_j, p_\rho)\theta(p_i, p_{i+1})k^i £(p_0, p_1))$

Consider $V_i = \prod_{j=0}^{i} \varphi(p_j, p_\rho)\theta(p_i, p_{i+1})k^i £(p_0, p_1))$

We have

(3.10) $V_{i+1}/ V_i = \varphi(p_{i+1}, p_\rho)\theta(p_{i+1}, p_{i+2})k / £(p_\sigma, p_\rho) \leq \theta(p_\sigma, p_{\sigma+1})£(p_\sigma, p_{\sigma+1})$

In view of condition 3.2 and ratio test, the series $\sum V_i$ converges. Thus $\lim_{\rho\to\infty} T_\sigma$ exists. Hence the real sequence $\{T_\sigma\}$ is Cauchy.

Now using 3.8 we get

(3.11) $£(p_\sigma, p_\rho) \leq £(p_0, p_1)[k^\sigma\theta(p_\sigma, p_{\sigma+1}) + (T_{\rho-1} - T\sigma)]$

We used $\theta(p, q) \geq 1$. Letting $\sigma,\rho \to \infty$in 3.11 then obtain

(3.12) $\lim_{\rho,\sigma\to\infty} £(p_\sigma, p_\rho) = 0$.

Thus the sequence $\{p_\sigma\}$ is Cauchy sequence in \$ and by the completeness of the space \$, we can affirm that some $p_\sigma \to p^*$ as $\sigma \to \infty$.

Now, we claim that p^* is an fixed point, From (3.1) , we have

(3.13) $£(p^*, p_{\sigma+1}) \leq \theta(p^*, p_\sigma)£(p^*, p_\sigma) + \varphi(p_{\sigma,}, p_{\sigma+1})£(p_{\sigma,}, p_{\sigma+1})$.

Knowing that the limit of $\theta(p^*, p_\sigma)$ and $\varphi(p_{\sigma,}, p_{\sigma+1})$ exists and are finite from (3.2) and using (3.12) we can affirm that

(3.14) $\lim_{\sigma\to\infty} £(p_\sigma, p^*) = 0$.

On the other hand, we have

$£(p^*, Tp^*) \le \theta(p^*, p_{\sigma+1})£(p^*, p_{\sigma+1}) + \varphi(p_{\sigma+1}, Tp^*)£(p_{\sigma+1}, Tp^*)$

$= \theta(p^*, p_{\sigma+1})£(p^*, p_{\sigma+1}) + \varphi(p_{\sigma+1}, Tp^*)£(Tp_\sigma, Tp^*)$

(3.15) $\le \theta(p^*, p_{\sigma+1})£(p^*, p_{\sigma+1}) + \varphi(p_{\sigma+1}, Tp^*)(h(p_\sigma) - h(p_{\sigma+1}))£(p_\sigma, p^*)$.

If we take the limit in (3.15) as $\sigma \to \infty$ and from (3.2) and (3.14) , we obtain

$£(p^*, Tp^*) = 0$,

That is p^* is a fixed point of T.

Assume that T has two fixed points p^* and p^{**} (that is, $Tp^* = p^*$ and $Tp^{**} = p^{**}$). Then

$$£(p^*, p^{**}) = £(Tp^*, Tp^{**}) \le (h(p^*) - h(Tp^*))\, £(p^*, p^{**})$$

$= (h(p^*) - h(p^*))\, £(p^*, p^{**}) = 0$.

Therefore, $£(p^*, p^{**}) = 0$ and $p^* = p^{**}$.

Example 2.2 [10]

Let \$ = {1. 2,3}. Let complex- valued double controlled metric -like £ : \$ x \$ → C defined by

$£(1, 1) = £(2, 2) = 0,\ £(3,3) = \frac{i}{2},\ £(1,2) = £(2,1) = 2+4i, £(2,3) = £(3,2) = i,\ £(1,3) = £(3, 1) = 1 - i$

And θ, φ: \$ x \$ → [1,+ ∞) to be symmetry and defined by

$\theta(1,1) = \theta(2,2) = \theta(3,3) = 1,\ \theta(1,2) = \theta(2,1) = \frac{6}{5},\ \theta(2,3) = \theta(3,2) = \frac{8}{5},\ \theta(1,3) = \theta(3,1) = \frac{151}{100}$.

$\varphi(1,1) = \varphi(3,3) = \varphi(2,2) = 1,\ \varphi(1,2) = \varphi(2,1) = \frac{6}{5},\ \varphi(2,3) = \varphi(3,2) = \frac{33}{20},\ \varphi(1,3) = \varphi(3,1) = \frac{8}{3}$

Now defined the self- mapping T on \$ as follows T(1) = T(2) = T(3) = 2,and h defined on \$ to R as h(1) = 6, h(2) = 5 and h(3) = 9.

Now verify the condition (3.1):

- ***Case 1:*** When $p = 1, q = 2$

  $|£(Tp, Tq)| = |£(T1, T2)| = |£(2, 2)| = 0 \le (h(1) - h(T1))\,|£(1, 2)|$

  $= (h(1) - h(2))\,|£(1, 2)|$

  $= 1.\sqrt{20} = \sqrt{20}$.

- ***Case 2:*** When $p = 1, q = 1$

  $|£(Tp, Tq)| = |£(T1, T1)| = |£(2, 2)| = 0 \le (h(1) - h(T1))\,|£(1, 1)|$

  $= (h(1) - h(2))\,|£(1, 1)|$

  $= 1.0 = 0$.

- ***Case 3:*** When p = 1 ,q = 3

  $|£(Tp, Tq)| = |£(T1, T3)| = |£(2, 2)| = 0 \leq (h(1) - h(T1))|£(1, 3)|$

  $= (h(1) - h(2))|£(1, 3)|$

  $= 1. \sqrt{2} = \sqrt{2}.$

- ***Case 4*:** When p = 2 ,q = 1

  $|£(Tp, Tq)| = |£(T2, T1)| = |£(2, 2)| = 0 \leq (h(2) - h(T2))|£(2, 1)|$

  $= (h(2) - h(2))|£(2, 1)|$

  $= 0.\sqrt{20} = 0.$

- ***Case* 5:** When p = 2 ,q = 2

  $|£(Tp, Tq)| = |£(T2, T2)| = |£(2, 2)| = 0 \leq (h(2) - h(T2))|£(2, 2)|$

  $= (h(2) - h(2))|£(2, 2)|$

  $= 0.0 = 0.$

- ***Case* 6:** When p = 2 ,q = 3

  $|£(Tp, Tq)| = |£(T2, T3.)| = |£(2, 2)| = 0 \leq (h(2) - h(T2))|£(2, 3)|$

  $= (h(2) - h(2))|£(2, 3)|$

  $= 0.1 = 0.$

- ***Case 7:*** When p = 3 ,q = 1

  $|£(Tp, Tq)| = |£(T3, T1)| = |£(2, 2)| = 0 \leq (h(3) - h(T3))|£(3, 1)|$

  $= (h(3) - h(2))|£(3, 1)|$

  $= 4.\sqrt{2}.$

- ***Case 8:*** When p = 3 ,q = 2

  $|£(Tp, Tq)| = |£(T3, T2)| = |£(2, 2)| = 0 \leq (h(3) - h(T3))|£(3, 2)|$

  $= (h(3) - h(2))|£(3, 2)|$

  $= 4. 1= 4.$

- ***Case 9:*** When p = 3 ,q = 3

  $|£(Tp, Tq)| = |£(T3, T3)| = |£(2, 2)| = 0 \leq (h(3) - h(T3))|£(3, 3)|$

  $= (h(3) - h(2))|£(3, 3)|$

  $= 4. 1/2= 2.$

For all k ε (0,1) , it is clear that the above conditions are satisfied, these conditions are also satisfied for T(1) = T(2) =T(3) = 1. For any p_0 ε $ condition (3.2) holds along with conditions of theorem 3.1. Therefore, there exists a unique fixed point at 1.

CONCLUSIONS

The results obtained in the setting of complex-valued double controlled metric-like spaces generalize those of Souayah and Hidri [15] and other related works

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